

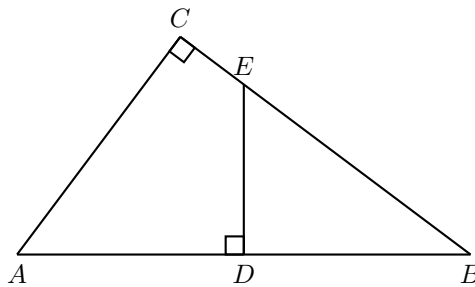
29th Annual Iowa Collegiate Mathematics Competition

Saturday, April 1, 2023

Problems written by Damiano Fulghesu, Minnesota State University Moorhead

*To receive full credit, all solutions require complete justification.
Calculators are allowed but not very helpful and certainly not necessary.
Books, notes, and other resources are prohibited.*

- Penny Game.** Aiden and Beatrix played a game of matching pennies. On each toss, Aiden won a penny from Beatrix if their coins matched, and Beatrix won one from Aiden if they failed to match. When they stopped, their coins had matched 13 times and Beatrix ended up with 8 more pennies than she started with. How many times did they toss?
- Straight Motion.** A particle is moving along a straight line so that its velocity at time t , in seconds, is $3t^2$ meters per second. At what time t during the interval from $t = 0$ seconds and $t = 9$ seconds is its velocity the same as its average velocity over the entire interval?
- Quadrilateral in a Triangle.** In the figure below, $AB = 20$, $AC = 12$, $AD = DB$, angles ACB and ADE are right angles. Determine the area of the quadrilateral $ADEC$.



- Same Quotient and Remainder.** When each of the numbers 887, 1242, and 2023 is divided by the integer $d > 1$, the same remainder r occurs. Determine the positive integers d and r .
- Exponential Equation.** Determine all real values of x satisfying the following equation:

$$4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}.$$

- Minimum Value.** Determine the minimum value of

$$\frac{x}{3y} + \frac{6y}{z} + \frac{4z}{x}$$

for positive real numbers x, y, z .

- Choose a Real Number.** Suppose that the real number x is chosen uniformly at random in the interval $(200, 300)$. Given that $\lfloor \sqrt{x} \rfloor = 15$, determine the probability that $\lfloor \sqrt{100x} \rfloor = 150$. Express your answer as a rational fraction in lowest terms.

Note. For any real number a , the notation $\lfloor a \rfloor$ represents the largest integer less than or equal to a .

- Roots of a Polynomial.** Let p, q , and r be distinct complex roots of the equation $x^3 - x^2 + x - 2 = 0$. Determine the value of $p^3 + q^3 + r^3$.

- A Functional Equation.** Determine all functions on the real numbers satisfying

$$f(1-x) + 2 = xf(x).$$

- Multiple Locks.** The door to a high security area is to have multiple locks. Each of the eleven persons is to receive an incomplete set of keys, subject to the following condition: Whenever six of the eleven are present, they have among them a key to every lock, but whenever fewer than six are present there is at least one of the locks that none of them can open. What is the smallest number of locks that will allow such a distribution of keys?