Twenty-third Annual Iowa Collegiate Mathematics Competition
University of Iowa, Saturday March 4, 2017

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To receive full credit, all solutions require complete justification.

PROBLEM 1. Solve for x.

Determine the set of all real numbers x satisfying the equation

$$|x - 2.6| + |x - 4.6| = 2.$$

PROBLEM 2. Recover blotted out digits.

The number 99 was multiplied by an integer k to obtain an integer of seven decimal digits , but two of the digits got blotted out on the paper. The product was $62ab427$, but the digits a and b are illegible. Determine all possible values of a and b.

PROBLEM 3. Two players and 2017 other persons.

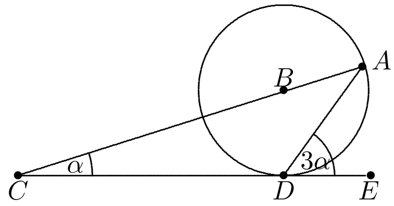
Two players A and B, and 2017 other persons, are arranged in a circle in such a way that A and B are not initially in adjacent positions. A and B play alternately, with A going first, and a play consists of choosing one of one's two immediate neighbors, who is then removed from the circle. The player (A or B) who removes the other player wins. Describe, with proof, a winning strategy for one of the players.

PROBLEM 4. A sum divisible by 11.

Each of the numbers $a_1, a_2, \ldots, a_{111}$ is a positive integer. Prove that among these numbers there are eleven numbers that have a sum divisible by 11.

PROBLEM 5. The measure of an angle.

In the figure at the right A and D are points on a circle centered at B. The line CE is tangent to the circle at D, and line CA passes through B. Angle ADE is three times angle ACE. Find the measure of angle ACE.



PROBLEM 6. Factoring 3^{2017}.

Determine the number of triples (x, y, z) of positive integers which satisfy $xyz = 3^{2017}$ and $x \le y \le z < x + y$.

PROBLEM 7. Roots in arithmetic progression.

Determine all real numbers m such that the equation

$$x^4 - (3m + 2)x^2 + m^2 = 0$$

has four real roots in arithmetic progression. (That four numbers a, b, c, d are in arithmetic progression means that $b - a = c - b = d - c$.)

PROBLEM 8. Bound for an integral.

Let $f\colon [0, \infty) \to$R be an everywhere differentiable function with f' monotone nondecreasing and $f(0) = 0$. Prove that for all x in $[0, \infty)$,

$$\int_0^x f(t)dt \le \frac{x^2}{2} f'(x).$$

PROBLEM 9. A minimum value.

Let r, s, t, u be real numbers. Prove that $\min\{r - s^2, s - t^2, t - u^2, u - r^2\} \le 1/4$.

Problem 10. Function value at 2017.

Let f be continuous on the reals to the reals, with $f(x) \cdot f(f(x)) = 1$ for all x. If $f(4034) = 4033$, find $f(2017)$.