Sixth Annual Iowa Collegiate Mathematics Competition

Luther College
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1. Let's Shake On It

A group of two hundred students, one hundred boys and one hundred girls, is randomly divided into two rows of 100 people each. Each person in one row is directly opposite a person in the other row, and the opposite pairs shake hands. Prove that, the number of "boy-boy" handshakes equals the number of "girl-girl" handshakes.

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2. I Know the Answer, What Was the Question?

On a multiple choice test one of the questions was illegible. The choice of answers was:
a) All of the below
b) None of the below
c) All of the above
d) Exactly one of the above
e) None of the above
f) None of the above

What was the correct answer?

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3. Factorial Zeros

Find the smallest positive integer N (or show that none exists) for which the decimal representation of N! ends in exactly two thousand zeros.

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4. Strips Can't Cover

Prove that a 10" x 10" board cannot be covered by twenty-five 1" x 4" strips.

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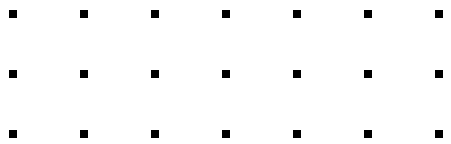
5. Connect the Dots



Each of the 21 dots in the above array is to be colored with one of two colors.
Prove that, no matter how the coloring is done, there will be four dots of the same color that form the vertices of a rectangle.

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6. It Strikes a Chord

Points A, B, C, and D are located counterclockwise around a circle and chord lengths $AB = 2$, $BC = 3$, $CD = 4$, and $DA = 6$. Find the diameter of the circle.

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7. Less Than Half Make More Than Half

Prove that if any set on nine distinct integers has sum greater than 200, then a subset of four of the integers has sum greater than 100.

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8. Fractional Inequality

Prove that $1/64 < (1/2)(3/4)(5/6)\ldots (1999/2000) < 1/44$

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9. As the Fly Flies

Al is riding his bicycle east along a straight path at 15 mph, Beth is 40 miles east of Al riding her bicycle west along the same path at 25 mph, and a fly, Phil, is at the front of Al's front tire flying east at 20 mph. When Phil reaches Beth's front tire, he immediately starts flying west towards Al at 30 mph, and when he reaches Al's front tire he immediately starts flying east toward Beth at 20 mph. Phil's back-and-forth flights continue until he meets his fate at the instant when Al and Beth's front tires meet.

On his ill-fated journey, did Phil fly farther traveling east or west? How much farther?

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10. Another Positive Polynomial

Let $P(x)$ be a polynomial of degree n, for which $P(x) \geq 0$ for all real numbers x.

Prove that $P(x) + P'(x) + P''(x) + \ldots + P^{(n)}(x) \geq 0$ for all real numbers x.

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